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Inflating p-branes

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Abstract: We look for solutions in Einstein gravity corresponding to inflating braneworlds of arbitrary dimension and co-dimension. These solutions correspond to isolated sources (no long range fields). Using dynamical systems techniques, we show that there exists a unique solution corresponding to a black p-brane with a regular horizon at the location of the brane. The solution is *not* however asymptotically flat, but has global deficit angles.

Keywords: Extra Large Dimensions, p-branes, Black Holes.

Contents

1. Introduction

Inflation is one of the key tenets of the modern standard cosmological model. It allegedly solves many of the problems of naturalness in the old hot big bang model, and perhaps it greatest allure is in 'predicting' a scale invariant perturbation spectrum — in ever increasing agreement with the observations of inhomogeneities in the microwave background [1]. It is however telling that no definitive inflationary model exists, indeed, no definitive observation of even a single scalar particle has yet been made, let alone a multitude, as many inflationary models require.

Although the issue of exponential expansion in the early universe is nominally an open question, current observations do seem to indicate that the universe is once more in a stage of gentle accelerated expansion [2]. Assuming that gravity is approximately four-dimensional and newtonian/einsteinian at large scales indicates that our universe has a negative equation of state at the current time [3]. This inevitability of an accelerating universe places strong demands on any underlying theory of fundamental physics — whether it be simply the challenge to produce an appropriate equation of state [4], or, more subtle questions about the consistency of a de-Sitter asymptotic state for the universe [5].

Braneworlds [6] are an interesting orthogonal development in the attempt to describe our universe within the context of a fundamental higher energy theory. The braneworld scenario imagines us as being confined to a four-dimensional hyperplane in a higher dimensional spacetime. Standard Model interactions and most usual phsyics is confined to the brane universe, with only gravity (or some small number of zero-mode fields) propagating in the bulk. Such a set-up can provide an interesting alternate solution to the hierarchy problem [7, 8], but it is crucial that such a framework is gravitationally and cosmologically consistent. Of course, on small scales we can allow KK graviton modes to be present, but on larger scales gravity must be einsteinian to be consistent with observation. Of course, on the extremely large scale, it is possible that gravity is not in fact newtonian (a possibility explored by Milgrom [9]), and in fact an unexpected bonus of some braneworld models is that gravity can be modified on large scales [10] in a generally covariant fashion.

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Within the context of Randall-Sundrum (RS) braneworlds [8, 11], a phenomenologically motivated model rather similar to heterotic M-theory compactifications [12], in which there is only one extra dimension, the description of our universe is particularly simple: Our universe is simply a hyperplane living in five negatively curved dimensions. Perturbative gravity is easily shown to be identical to perturbative four dimensional Einstein gravity [11, 13], and in the pure RS set-up where only gravity propagates in the bulk, the full set of solutions for our cosmological braneworld are known and easily found as moving branes in a Schwarzschild anti-de Sitter (adS) bulk [14]. In particular, the standard inflationary universe is simply a uniformly moving brane in pure adS spacetime.

In string theory however, we live in more than five dimensions, and even in the heterotic M-theory compactification [15], the universe is only effectively five-dimensional in a small range of high energies. The more string-motivated Arkani-Hamed et al. compactifications [7] have many extra dimensions. However, many of these models do not include the gravitational effect of the energy-momentum of the brane itself. One of the reasons that the RS models are so easy to deal with is the fact that gravity in one spatial dimension is trivial (recall that the universe is homogeneous and isotropic to leading order), and this renders the problem of finding cosmological solutions straightforward. With more extra dimensions however, the effect of consistently including the stress-energy of the brane becomes nontrivial. If we have three or more extra spatial dimensions, local warping of those extra dimensions induced by the stress-energy of the braneworld creates a naked null singularity at the putative brane [16], which although it does not stop physics in the bulk being well-defined, does mean that any physical description of the brane is highly dependent on the way the brane is modelled [17]. (A particularly interesting variant being the nonsingular "blown-up" p-brane [18].)

Given the natural interest in inflationary solutions for our universe, and the interest in finding braneworld resolutions to various cosmological problems, it is an obvious question to try to find general inflating braneworld solutions. Of course, these are well known for the RS case of codimension one. However, inflating braneworlds for higher codimension are not explicitly known. (Although a recent paper by Olasagasti and Tamvakis [19] looks for inflating solutions exterior to a global defect, extended to include the core by Cho and Vilenkin [20].) The existence of asymptotically flat solutions has been assumed in work attempting to incorporate stringy inflation via brane motion on the compactification manifold [21], and the existence of regular solutions assumed in work seeking a self-tuning mechanism for the small value of the cosmological constant [22]. The fact remains however, that there are no isolated inflating braneworld-type solutions known for codimension three or higher.

To understand why this might be a nontrivial question, consider the marginal case of codimension two. In many six-dimensional braneworld models, our brane appears as a conical deficit with an induced Minkowski flat metric. This is easily modelled within field theory as a 'cosmic string' defect, just as the RS model is a domain wall. However, there are two sorts of field theory vortex — the local vortex, which has a conical deficit as above, or the global, which has a long-range Goldstone boson field, and is not simply an isolated conical deficit. The gravitational effect of this long-range field is to cause a self-

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compactification of the spacetime [23] in a manner similar to that of the vacuum domain wall [24], and the induced metric of our braneworld is in fact a de Sitter universe [25] again like that of the domain wall [26]. If a Minkowski braneworld metric is desired, then it is necessary to introduce a negative cosmological constant [27], and one recovers a hierarchy resolving RS-style model [27, 28]. The interpretation of this gravitational interaction is that if one forces a braneworld to have a particular induced metric, say Minkowski, then the solution of the gravitational equations will generically be singular at a finite distance from the braneworld [28, 29]. However, if one tunes a bulk cosmological constant against a braneworld Hubble expansion, then there is a one parameter family $H(\Lambda)$ for which a nonsingular solution exists [23]. This would appear to be a general result, not just confined to the global vortex in Einstein gravity, as Berglund et al. [30] found a similar behaviour within low energy string gravity while looking for inflating codimension two solutions in an attempt to incorporate de Sitter space into string theory in a natural way. It is tempting to conjecture that the singularity of the self-gravitating cosmic p-brane [16] could also be resolved by a similar process, however, there are two key differences with the global vortex: The p-brane singularity lies at its core, rather than at finite distance, and is asymptotically flat — i.e. not compact. There is also the lack of energy-momentum in the bulk, since there is no long range Goldstone field. It is therefore not at all clear that allowing the brane to inflate will solve the problem of the singularity at the core — or that if it does, another singularity might not appear at finite distance.

Returning to the isolated codimension two inflating brane, note that this will be a solution of the Einstein equations where the metric can be chosen to take the form

$$ds^2 = A^2(r)\left[dt^2 - \cosh^2 t\, d\Omega^2_{D-3}\right] - B^{-2}(r)dr^2 - B^2(r)d\theta^2 \,. \tag{1.1}$$

Fortunately, we do not need to actually write down the Einstein equations and solve them, since if we double analytically continue this metric by making θ a time coordinate, and $t \to i\chi + \pi/2$ a spacelike coordinate, this is readily seen to be a spherically symmetric static solution in D dimensions, and is hence the Schwarzschild solution. Thus

$$ds^2 = r^2\left[dt^2 - \cosh^2 t\, d\Omega^2_{D-3}\right] - \left(1 - \left(\frac{r_+}{r}\right)^{D-3}\right)^{-1} dr^2 - \left(1 - \left(\frac{r_+}{r}\right)^{D-3}\right) d\theta^2 \,. \tag{1.2}$$

Although the transverse (braneworld) dimensions are those of a lorentzian inflating universe, this is easily seen to have the (r, θ) geometry of the euclidean black hole 'cigar'. As such, $r = r_+$ can be thought of as the location of the brane. In fact, the metric (1.2) with $D = 5$ was used by Witten [31] to demonstrate an instability of the KK vacuum.

Conventionally, in black hole thermodynamics, the periodicity of euclidean time (here the θ-angle) is fixed by requiring regularity at $r = r_+$, however, if we are looking for a solution corresponding to a codimension two brane, we do not want regularity, rather, it is precisely the conical deficit at $r = r_+$ that will indicate the presence of the brane. Just as with the standard cosmic string, there should be a deficit of $8\pi\mu$ where μ is the energy per unit p-area of the brane in Planck units. Computing the metric near $r = r_+$ gives the relation between the periodicity of the θ-angle, the energy of the brane, and r_+ as:

$$\delta\theta = \frac{4\pi r_+}{D-3}\,(1 - 4G\mu)\,. \tag{1.3}$$

As $r \to \infty$, the metric is asymptotically the KK vacuum $\mathbf{R}_{p+2} \times S^1$, written in Rindler coordinates ($\mathbf{X}_{p+1} = r \cosh t \ \mathbf{n}_{p+1}$, $T = r \sinh t$), with the internal circle having dimension $(D-3)/[2r_+(1-4G\mu)]$.

Although this solution is regular, it is different in character from the inflating wall solution, which is simply a moving brane in some five-dimensional background bulk. Here, the bulk is necessarily the KK vacuum, rather than being noncompact, and a *'bubble of nothing'* is present in the spacetime. If already the spacetime of an inflating codimension two brane is so phenomenologically different from codimension one, we cannot expect to use intuition to deduce what higher codimension inflating branes will look like. We must therefore actually search for solutions, which is what we will now do. We first derive the Einstein equations, and revisit the Poincaré invariant p-branes of reference [16] in the context of a dynamical system. Then we analyse the inflating brane solutions. We comment on branes with anti-de Sitter geometries before concluding.

2. Gravitational equations

In general, a p-brane produced by some localized source with energy and tension of the same magnitude need not be Poincaré invariant, but can in fact have an induced metric which is constant curvature. For an inflating brane, this will be constant positive curvature. We therefore look for a solution which is a warped product of this constant curvature worldbrane metric, and dependent on some orthogonal coordinate. The metric can be written as:

$$ds^2 = A^2(r)\left[dt^2 - \cosh^2(\sqrt{\kappa}t)\, d\Omega_p'^2\right] - B^2(r)dr^2 - C^2(r)d\Omega_n^2\,, \tag{2.1}$$

where κ has been added explicitly for comparison with the standard Poincaré invariant cosmic p-branes. Note this metric has brane dimension $p+1$ and codimension $n+1$.

The Einstein equations for this metric are:

$$R^t_t = \frac{1}{B^2}\left[\frac{A''}{A} - \frac{A'B'}{AB} + p\frac{A'^2}{A^2} + n\frac{A'C'}{AC}\right] - \frac{p\kappa}{A^2} \tag{2.2}$$

$$R^r_r = \frac{(p+1)}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right) + \frac{n}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right) \tag{2.3}$$

$$R^\theta_\theta = \frac{1}{B^2}\left[\frac{C''}{C} - \frac{C'B'}{CB} + (p+1)\frac{A'C'}{AC} + (n-1)\frac{C'^2}{C^2}\right] - \frac{(n-1)}{C^2}\,. \tag{2.4}$$

At this point, it is immediate that there is no possibility of a *'bubble of nothing'* type of solution to the higher codimension brane with $A \sim r$, and B, C roughly constant, since (2.4) cannot satisfy the Einstein equation $R^\theta_\theta = 0$.

In the case of the Poincaré invariant brane, where there is no κ term in (2.2), the Einstein equations can be directly integrated for a suitable choice of the function B, giving the cosmic p-branes [16]

$$ds^2 = \left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right)^a \left[dt^2 - dy_p^2\right] - \left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right)^b dr^2 - r^2\left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right)^c d\Omega_n^2\,, \tag{2.5}$$

where

$$a = \frac{\sqrt{n}}{\sqrt{(n+p)(p+1)}}, \qquad b = -\frac{[(n-2) + a(p+1)]}{(n-1)}, \qquad c = 1 + b. \tag{2.6}$$

For the inflating brane we have not been able to find an exact analytic solution, however, by re-expressing the Einstein equations as a two-dimensional dynamical system, it is possible to demonstrate the existence of a solution, and to derive its general form.

To do this, let $B \equiv A$ and define

$$X = \frac{A'}{A} + \frac{n}{p}\frac{C'}{C} \tag{2.7}$$

$$Y = \frac{C'}{C} \tag{2.8}$$

in which case (2.2)–(2.4) can be rewritten as

$$X' = X^2 - \kappa - \frac{n(n+p)}{p^2} Y^2 \tag{2.9}$$

$$Y' = \frac{p(p+1)}{n}(X^2 - \kappa) - pXY - \frac{(n+p)}{p} Y^2 \tag{2.10}$$

together with the constraint

$$\mathcal{C}(X,Y) = p(p+1)(X^2 - \kappa) - \frac{n(n+p)}{p} Y^2 = n(n-1)\frac{A^2}{C^2}. \tag{2.11}$$

Note that

$$\frac{d\mathcal{C}}{dr} = 2\mathcal{C}\left[X - \frac{(n+p)}{p} Y\right] \tag{2.12}$$

hence $\mathcal{C} = 0$ represents an invariant hyperboloid in the phase plane.

In addition, there are two pairs of critical points

$$P_\pm = \pm\sqrt{\kappa}\,(1,0)$$
$$Q_\pm = \pm\sqrt{\kappa}\left(\sqrt{\frac{n+p}{p}}, \sqrt{\frac{p}{n+p}}\right) \tag{2.13}$$

for $\kappa = 1$, which merge to a single critical point at the origin for the Poincaré brane, $\kappa = 0$.

Although we will give a more detailed analysis of the inflating brane phase plane in the next section, the main features to note at this stage are that $P_\pm$ lie on the invariant hyperboloid $\mathcal{C} = 0$, whereas the $Q_\pm$ have $\mathcal{C} = np\kappa > 0$; both are therefore in the physically allowed region of the phase plane $\mathcal{C} \geq 0$. The $P_\pm$ are saddle points, but the precise nature of the $Q_\pm$ critical points depends on the overall dimensionality of spacetime. In general Q_+ is an attractor, and Q_- is a repeller, however, if $D < 10$, these critical points are foci, hence trajectories approach or repel in a vortical fashion.

Before turning to the inflating brane solutions, it is actually useful to first analyse the Poincaré phase plane, since in this case we actually have the metric explicitly, and can therefore directly calculate X and Y. The phase plane is given by solving (2.9) and (2.10) with $\kappa = 0$. As already mentioned, the critical points merge into a single degenerate critical

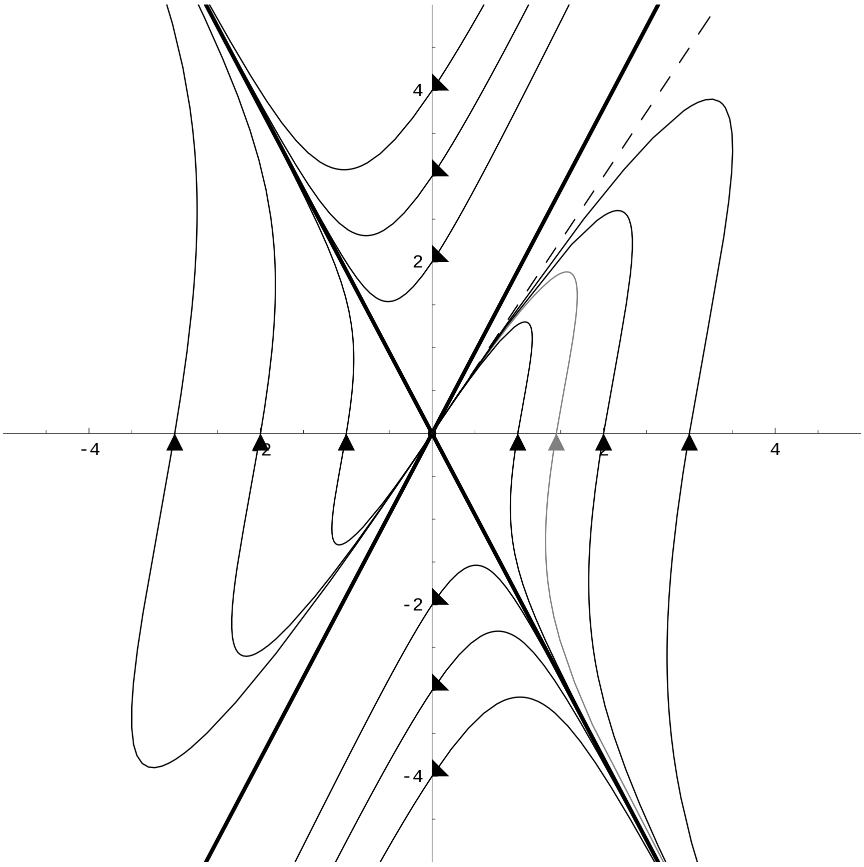

Figure 1: The phase plane of the Poincaré p-brane. The invariant curve $\mathcal{C} = 0$ is the pair of thick stright lines, and the Minkowski spacetime solution is the dotted line. A generic p-brane solution is shown in grey.

point at the origin: $\mathbf{P} = (0, 0)$. Clearly the invariant curve $\mathcal{C}$ is now a pair of straight lines $Y = \pm\gamma^{-1}X$, where we have defined

$$\gamma = \frac{\sqrt{n(n+p)/(p+1)}}{p} \tag{2.14}$$

for later use. The phase plane is shown in figure 1 for the same values $n = 2$ and $p = 3$ for comparison with figure 2.

Notice that a typical trajectory in the right hand quadrant starts off at large $(X, -Y)$, asymptoting $\mathcal{C} = 0$, curves around, and approaches $\mathbf{P}$ tangent to the dotted line $Y = pX/n$ in general. Solving for the asymptotic small r region gives

$$X = \frac{1}{pr}, \qquad Y = -\frac{1}{p\gamma r} \tag{2.15}$$

which in turn gives the solution

$$A \propto r^{\frac{1}{p} + \frac{n}{p^2\gamma}}, \qquad C \propto r^{-1/p\gamma} \tag{2.16}$$

recalling that $B = A$ in these coordinates, and transforming to the radial coordinate in which the p-brane metric (2.5) takes its canonical form ($A^{-2}dr_p = Adr$) indeed shows that this is the "near-core" régime $r \to r_+$ of the p-brane (which of course is a null singularity). For $\{X, Y\} \to \mathbf{P}$ on the other hand, we have

$$A \sim 1 - \frac{\alpha_0}{r^{n-1}}\,, \qquad C = r\left(1 + \frac{(p+2)\alpha_0}{(n-2)r^{n-1}}\right) \tag{2.17}$$

the asymptotic far-field régime of the p-brane solution.

Specifically, the exact form of the Poincaré p-brane solution (2.5) gives

$$\begin{aligned} X &= \frac{2u}{3r_+}(1-u)^{\sqrt{\frac{5}{8}}-1}\left[1 - \frac{(\sqrt{5}+2\sqrt{2})}{4\sqrt{2}}u\right] \\ Y &= \frac{u}{r_+}(1-u)^{\sqrt{\frac{5}{8}}-1}\left[1 - \frac{(\sqrt{5}+2\sqrt{2})}{2\sqrt{5}}u\right]. \end{aligned} \tag{2.18}$$

We see therefore that altering the mass of the solution simply scales the plot in the phase plane. A representative trajectory with $r_+ = 4/30$ is shown in grey in figure 1. Note that for $r_+ = 0$, i.e. flat space, we have $Y = pX/n$ (shown as a dotted line), which is a separatrix in the Poincaré phase plane. This also shows manifestly that the solutions are asymptotically flat.

3. Inflating branes and their global structure

Now turn to the inflating brane phase plane. A phase plot of this system for the values $p = 3$, $n = 2$ is shown in figure 2. For large (X, Y), the system asymptotes the Poincaré plane, therefore we expect the physical solutions to correspond to trajectories in the right hand exterior of the invariant hyperboloid. Indeed, comparing figure 2 to figure 1, we spot that there are a similar family of trajectories asymptoting the invariant hyperboloid for large $X \propto -Y$ which now terminate on Q_+. However, there are now some additional interesting solutions. The splitting of $\mathbf{P}$ into the two pairs of critical points allows a single trajectory from P_+ to Q_+. Also of later use is the existence of the other stable manifold trajectory connecting large negative $X \propto Y$ to P_+.

Dealing with the typical trajectory first, we note that these are analogous to the Poincaré curves - they asymptote the near singularity régime of (2.5). However, unlike the Poincaré solutions, these cannot have an asymptotically flat solution. Apart from $\mathcal{C} = 0$ itself, all of these trajectories terminate on Q_+. An analysis of Q_+ shows that the metric in this asymptotic region is

$$ds^2 \simeq \frac{p}{(p+n)}\rho^2\left[dt^2 - \cosh^2 t\, d\Omega_p^2\right] - d\rho^2 - \frac{(n-1)}{(n+p)}\rho^2 d\Omega_n^2 \tag{3.1}$$

as $\rho \to \infty$. This latter metric, while asymptotically locally flat, is not asymptotically flat, as it has global deficit angles in the spatial S^n part of the metric, as well as in the inflating braneworld part.

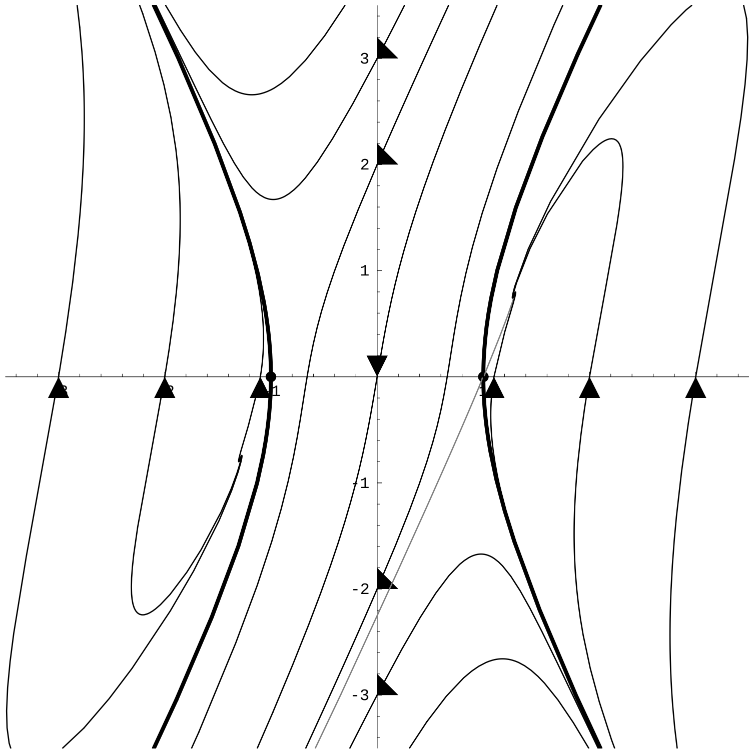

Figure 2: The inflating brane phase plane. The invariant hyperboloid $\mathcal{C} = 0$ is shown in bold again, the critical points by a dot, and the candidate trajectory from P_+ to Q_+ in grey (as well as its continuation into the interior-horizon region).

The net result is that apart from this asymptotic global deficit angle, the metric of these solutions is somewhat similar to the Poincaré p-brane, in that it has a null naked singularity as a near-field limit, which has an infinite area. This singularity integrates out to an ALF spacetime with a global deficit angle. Presumably, as with the standard p-brane [17], propagators are well defined on this spacetime, although we have not explored this issue.

One might therefore think that the inflating brane spacetime is similar to that of the Poincaré brane, however, there is one other possibility not present in the Poincaré phase plane, illustrated as the grey trajectory in figure 2, and that is the trajectory from P_+ to Q_+. Analysing the spacetime near $P_\pm$ shows that these critical points correspond to horizons; in a suitable coordinate system, the metric for the solution near P_+ is

$$ds^2 \simeq \rho^2 \left(1 - \frac{n(n-1)}{3(p+1)(p+2)C_0^2}\rho^2\right) dx_{p+1}^2 - d\rho^2 - C_0^2 \left(1 + \frac{(n-1)\rho^2}{(p+2)C_0^2}\right) d\Omega_n^2 \qquad (3.2)$$

as $\rho \to 0$.

To see this is a simple horizon, and also to obtain the maximal analytic extension of the spacetime, return to the phase plane coordinate $r = -\ln\rho$, and let

$$U = e^{t-r} \qquad , \qquad V = -e^{-t-r} \qquad (3.3)$$

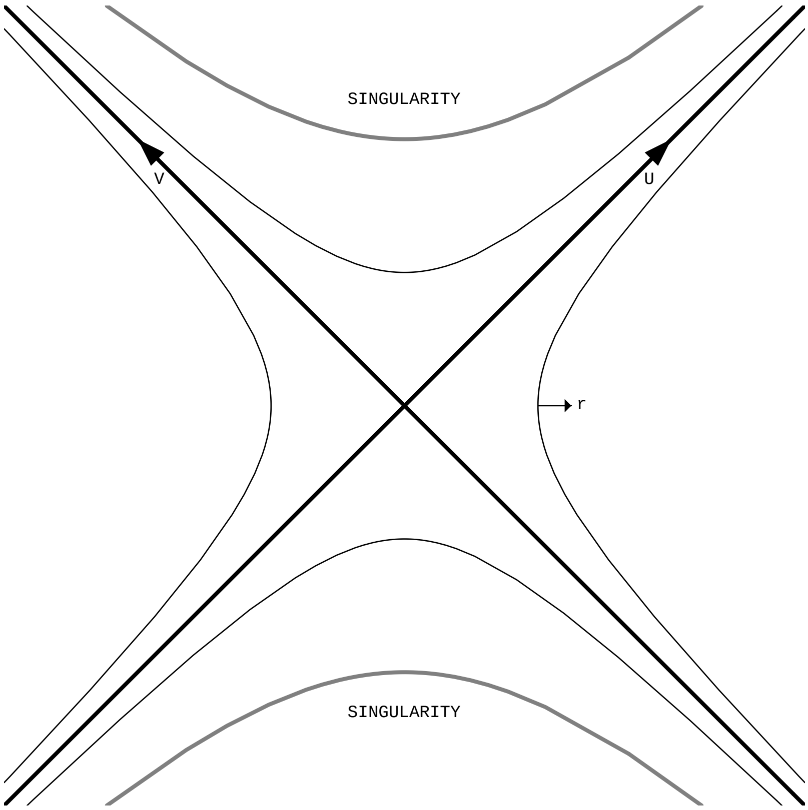

Figure 3: The Kruskal diagram of the inflating brane. The horizon (shown as the thick lines) is represented by the critical point P_+. The r-coordinate is shown in the exterior region, and the singularity as a grey line.

in which the near P_+ metric (3.2) is

$$ds^2 \sim dUdV - \frac{(U-V)^2}{4} d\Omega_p^2 - C_0^2 d\Omega_n^2 \tag{3.4}$$

as with conventional Kruskal coordinates (see figure 3)

Finally, defining

$$U = e^{\xi - \tau}\,, \qquad V = e^{-\xi-\tau} \tag{3.5}$$

gives

$$ds^2 \sim e^{-2\tau}\left[d\tau^2 - d\xi^2 - \sinh^2\xi d\Omega_p^2\right] - C_0^2 d\Omega_n^2 \tag{3.6}$$

as the analytically continued metric just interior to the horizon.

This shows that the extension across the horizon is (cf. the global vortex [23])

$$ds^2 = \bar{B}^2(\tau)d\tau^2 - \bar{A}^2(\tau)dH_{p+1}^2 - \bar{C}^2(\tau)d\Omega_n^2\,, \tag{3.7}$$

where dH_{p+1}^2 is the metric on a unit $(p+1)$-dimensional hyperbolic space. This is of course a time dependent metric as one would expect for an interior horizon régime.

Following through the computation of the Einstein equations and setting $\bar{B} = \bar{A}$, we once again obtain (2.9) and (2.10) as the dynamical system for the interior horizon where prime now denotes $d/d\tau$, and X and Y are defined in terms of the barred variables. The only difference is that the constraint (2.11) now reads

$$\mathcal{C}(X,Y) = -n(n-1)\frac{\bar{A}^2}{\bar{C}^2}\,. \tag{3.8}$$

i.e. the interior horizon régime corresponds to the connected region of the phase plane between the two branches of the invariant hyperboloid. It is not difficult to verify that the trajectory from P_+ to large negative X and Y in fact corresponds to an interior horizon solution terminating on a spacelike singularity.

This shows that the causal structure of the inflating p-brane is indeed given by figure 3, and the inflating brane is now a genuine black hole with an horizon. Of course, because we have only derived general properties of the solution, and particular asymptotic forms, we do not know the precise value of the mass of the black hole. Indeed, defining the mass of such an ALF spacetime is problematic [32].

4. Discussion

For completeness we would like to remark on the adS p-braneworlds. These are braneworlds in which the metric is a warped product of an anti de-Sitter braneworld with an $n+1$-dimensional orthogonal space, and correspond to the dynamical system (2.9) and (2.10) with $\kappa = -1$. For this value of κ there are no finite critical points on the phase plane, and the invariant hyperboloid $\mathcal{C} = 0$ now changes from being 'timelike' to 'spacelike' in the $X-Y$ plane. The typical trajectory now asymptotes the invariant hyperboloid at both ends, i.e. the solution has a null singularity both at the 'core' of the brane, and at finite radial distance. Vacuum adS branes are therefore generally singular.

This result seems in keeping with the general pattern for the global vortex, [25], where one could prove that the metric was singular for Poincaré or adS branes, but that for a de Sitter geometry on the brane with a fine tuned Hubble constant related to the brane tension, there was a nonsingular solution. Here there will be a similar tuning, since (2.1) has explicitly set the Hubble constant $H = 1$. We can reintroduce it at the expense of rescaling r, and hence the phase plane.

Of course, this general similarity with the global vortex then begs the question of whether we can remove the singularity of the Poincaré brane by adding a negative cosmological constant in the bulk as with the global vortex RS compactification [27]. However, a quick look at (2.2) shows that this is not possible. For codimension three or higher, we expect that the null singularity of the p-brane would be smoothed into a horizon coordinate singularity, i.e. $A \sim \rho$, $B = 1$, $C \sim C_0$ for $\rho \to 0$. This is clearly not compatible with (2.2).

We can now return to some of the potential applications of these inflating solutions and remark on whether they seem consistent with the assumptions made. First of all, if one wishes to place branes on compact manifolds, the issue of the global deficit angles must be addressed. Of course, these are uncharged branes, however, in [16] it was shown that the general properties of the uncharged spacetime were maintained when charge and even a dilaton with arbitrary coupling was also included. The only nonsingular spacetime in that

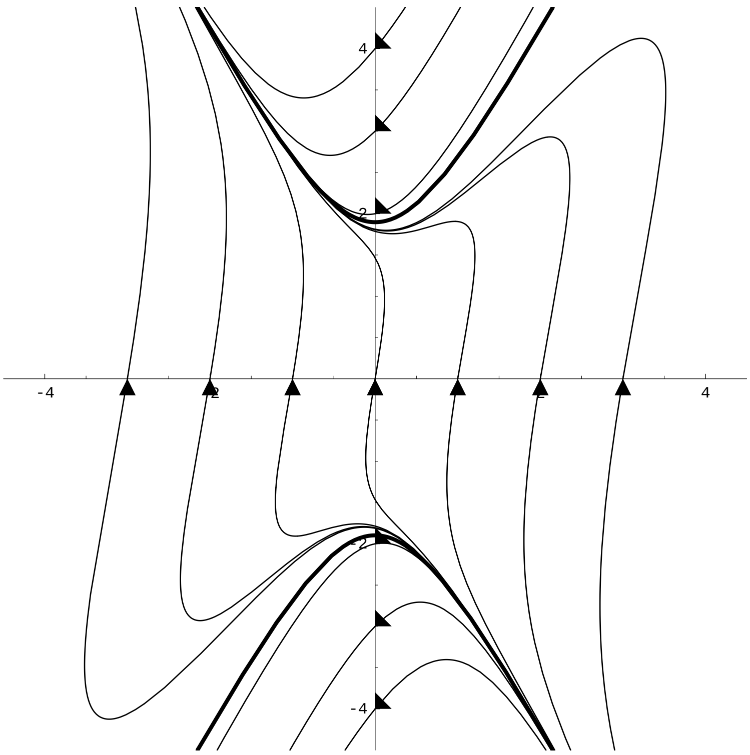

Figure 4: The adS brane phase plane. The invariant hyperboloid $\mathcal{C} = 0$ is shown in bold again, but now there are no critical points.

case was the extremally charged one. Based on general expectation, and on the results of Berglund et al. [30], these would be expected to actually become singular if one attempted to introduce inflation on the brane.

The other interesting application of inflating brane solutions is in a possible answer to the smallness of the cosmological constant [22]. In these papers, the authors supposed that introducing an expansion on the brane would smooth out the null singularity. To some extent we have backed up this assumption, however, this solution is not asymptotically flat, and it is not clear how these different asymptotics would affect gravity on the brane. Another key issue is that the authors claim that the Hubble constant is *inversely* proportional to the mass of the brane — a claim queried by Cho and Vilenkin [20]. Questions of how to define the mass of the brane notwithstanding, we cannot directly comment on this issue in the absence of an actual solution which would directly interpolate between the near-horizon solution (3.2) and the asymptotic spacetime (3.1). However, it is interesting to note that increasing the mass of the Poincaré solution actually shrinks the phase plane, in other words, the trajectories in figure 1 with higher mass are those *closer* to the origin. Similarly, introducing an explicit Hubble expansion on the brane changes the scale of figure 2 moving the critical points $P_\pm$ to $(\pm H, 0)$. Therefore decreasing H also shrinks the phase plane. Curiously therefore, this scaling does not seem to contradict the claims of Dvali et al., however, the argument is extremely unreliable given that r_+ tracks a genuine ADM mass in the Poincaré solution, and the thorny issue of mass in the inflating brane solution needs to be resolved.

To sum up, the pure gravitating p-brane can have a nonsingular (exterior to the horizon) geometry in which the induced metric parallel to the brane is an inflating de Sitter universe. The metric is ALF, but has global deficit angles, and in the absence of a core model for the brane has a black hole horizon. The presence of the regular horizon, as well as $g_{\theta\theta}$ being monotonic for this solution, indicates that we can replace this horizon by a general core model in an analogous fashion to the replacement of the Reissner-Nordstrom horizon by an SU(2) monopole core [33].

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